Mathematical targets and personal autonomy

Anna Contardi, Michele Pertichino and Brunetto Piochi

Italian Association for Down Persons, Rome; Interuniversity Department of Mathematics, Bari; Department of Mathematics "U. Dini", University of Florence

This article explains how mathematical skills can foster independence for daily living and how we should always have high expectations for children with Down syndrome.

A learning path about logics and mathematics for children with severe learning difficulties is too often considered extremely difficult, or even impossible. Even though these children very frequently have difficulties in abstracting, we do not think this is a reason for giving up; we rather believe that this awareness should stimulate a different way of teaching. Without denying or undervaluing the hindrances, we actually believe that it is necessary to support the right of every child to learn as much as possible, even in the field of mathematics, as some good mathematical knowledge is an essential requirement to gain autonomy in life.

Social autonomy

The achievement of autonomy is a fundamental aim for the growth and the social inclusion of any person, even if they are disabled.

Italy has chosen full inclusion in school for children with disabilities. The outline law for the assistance, social inclusion and the rights of disabled persons (law 5/2/92 n. 104) asserts that "mainstream school aims at developing the potentialities of the disabled person in learning, communication, relationships and socialisation". This concerns all subjects, since all of them are essential for the ripening of the individual and the accomplishment of the greatest autonomy for everyone.

What does being autonomous mean?

The answer to this question can be first answered in terms of capacity and behaviours:

- to know how to take care of oneself and one's living places
- to know how to communicate (in different ways and by different means)
- to be able to get one's bearings
- to know how to spend money
- to know how to use public services
- to know how to ask for help, etc.

The concept of autonomy we are referring to here is meant in its general sense, either as the capacity to observe and the awareness of one's skills and limitations, or as the ability to move in the external world and to actively get in touch with other people or things: "Autonomy is not doing everything on one's own. On the contrary it is knowing how to cooperate, ask and put things together".[1]

Even though few people are aware of it, the knowledge of some mathematical concepts is an essential requirement for the development of autonomy, either in terms of behaviours or in a more general sense. The role played by mathematics in becoming autonomous is clearly pointed out if we consider that "Mathematical education contributes […] to form the necessary abilities in order to interpret (reality) in a critical way and to intervene in it with awareness" (Italian Syllabus for Elementary School, 1985).

Children, including those with disabilities, have rights for example to autonomy, socialisation and an understanding of their culture. Schools are partly responsible for ensuring that children's rights are upheld. School is at present the main place, and often the only one, where children with disabilities have those rights upheld. In some countries, such as Italy, this happens at school by means of inclusion, and as far as possible this should concern the whole range of subjects, since all of them are important for the global development of a person.

Mathematical education must contribute to the cultural development of the citizen, in order to allow him or her to consciously take part in social life, showing critical ability. Starting from concrete experiences for the pupil, the teaching of mathematics must gradually initiate the use of language and mathematical reasoning as useful instruments to look at reality.[see also 2]

This motivates the choice of training to the study of mathematics by means of aims and activities directly related to the reality of life and therefore to the achievement of autonomy. We believe that this option should be the discriminating principle for the choice of mathematical teaching methods and contents for disadvantaged pupils. Such an option makes possible a coherent use of activities and for developing and evaluating mathematical skills in the context of autonomy.

Please note that mathematical concepts are also present in activities which are generally thought

 ISSN: 1463-6212
http://www.down-syndrome.info/library/periodicals/dsnu/04/01/

as non-mathematical. Some trivial examples: recognizing, naming and classifying objects, even if following various and imaginative principles, are main mathematical aims; crossing a street (apart from possible coordination and motion problems) requires estimating distances, speed, orientation and direction; screwing in a screw or a light bulb implies the concepts of horizontal-vertical rotation and direction, and requires estimates of length and width.

Crossing a street (apart from possible coordination and motion problems) requires estimating distances, speed, orientation and direction; screwing in a screw or a light bulb implies the concepts of horizontal-vertical rotation and direction, and requires estimates of length and width.

A lot of mathematical abilities can be attained through other non-mathematical activities, related to language, science or some practical applications. We find it important to be aware of the mathematical value of these activities; when teaching children with intellectual disabilities, as often it will be enough for the child to have a practical working knowledge of the concepts connected to their use when they are applied, and no explicit theoretical knowledge will be required.

A problem-solving approach

When thinking of mathematics, we must always recall that "[mathematical] theories derive and rise from several problems, and the concepts are developed on the questions that they have to solve, and the reasonings they intervene in".

The central role assigned to problem solving by mathematical school syllabi of many countries actually suggests starting from the solution of concrete problems in order to acquire mathematical concepts. This kind of approach to mathematics is certainly closer to the learning characteristics of pupils with disabilities.[see 4] But, if one agrees that "solving problems is a specific task of intelligence and intelligence is the specific gift of the human race",[5] then we believe that exploring and solving problems is a fundamental activity *for all students* in order to master concepts and abilities. The challenge introduced by a problem introduces the best way to stimulate the intelligence of every child and to help in developing their skills.

We believe that exploring and solving problems is a fundamental activity* for all students *in order to master concepts and abilities.

At present, the term "problem" has a shifting meaning in everyday school praxis. School books usually use this word to denote standard exercises, very often similar to each other and mass-produced. On the contrary, following the classical gestalt definition by Duncker,[6] a problem arises when a living being, willing to reach a goal, cannot do it by means of instinct or by an acquired behaviour.

In the following, when talking of *problems* we mean "The presence of a reason in the problematic situation and of a hindrance which does not allow for a direct action will create derangement and tension in the cognitive field of an individual, inciting him to do something to restore the balance";[7] whereas we will use exercises to denote "word problems" as presented in school books.

Exercises and practice

We are convinced of the great role of exercises in school life: they allow the reinforcement and the automating of techniques, but they are not the right means to work on concepts. It is well shown how useless and unreasonable the simple repetition of exercises can be, in order to promote a mechanical learning, if the abstraction capabilities of the person are not considered. This kind of practice has indeed turned out to be ineffective for the applied use of skills learnt by rote and for medium or long-term use, mainly because of limitations of short-term memory in pupils with learning difficulties. Exercises are absolutely necessary to strengthen and reinforce learning, to turn it into automatism, yet a meaningful achievement of learning must occur in another way, that is involving the child in autonomous discovery.

Problem solving

In contrast, problem-solving stimulates and gives meaning to skills, above all when it comes from concrete situations, at the same time stimulating attention, the use of previously acquired competences and the demand for collaboration amongst pupils, even in emotionally involving situations. The suggestion of a problem implies the stimulation and the interest of a person, it issues a challenge, inciting towards personal research in which knowledge is used to produce a new solution.

In this case the pupil will draw on all his gifts and abilities (how many and whatever they are) to go beyond the difficulties of the problem. The deficiency is then no more a hindrance: when facing a problem everyone has got to react exactly in the same way others do, whichever knowledge level he or she has. We must not forget that a problem rules out by its very nature the immediate solution, the quick answer; there is a problem when it is necessary to work on the request to reach a solution.

Of course, a question cannot be a problem for everyone equally; it may be a problem for some people but simply be a more or less difficult exercise for others. It is the first-person involvement that stirs up the personal necessity to solve a problem. This involvement allows the person to go beyond any intrinsic difficulty and to get to a conclusion. This is even more true in the case of people with developmental disabilities, where these difficulties are greater and the subjective stimulation to the solution is therefore essential.

Making learning fun

As for a person with disabilities, it is necessary to pay attention to both the cognitive and the affective-emotional fields. The first requires a special simplification of the passages in didactical suggestions to increase their intelligibility, together with an extremely concrete and effective approach; the latter calls for a choice of involving and reassuring situations and methods, to help the child to be confident of his own possibilities and to do his best. This function will be played by the choice for the setting of activities; playful situations and materials connected to ordinary life (the house, the meal, the toys, etc.) or recurring tales in common practice, therefore well known by children.

For the teacher ...

At last, we want to point out that by means of problem solving activities a teacher can know the learning level

 ISSN: 1463-6212
http://www.down-syndrome.info/library/periodicals/dsnu/04/01/

of every pupil, using this method to identify suitable learning targets. Moreover, problem solving gives us the chance to put into practice a different kind of working, leading to a better reassessment of knowing by means of tries and mistakes, in a collaborative, not competitive environment.

Such an organization of work within the classroom gives the child with disabilities better opportunities to socialize, but requires a more creative teaching method. During the same activity, it must be possible to make different requests to the disabled and to typically developing pupils, and among the latter as well, in order to meet individual needs or turn the gifts of the single individual to better account. Besides, when working in this way the pupil with disabilities can carry out many activities together with the whole class. There will certainly be moments of individual reinforcement of skills spent alone with the remedial teacher or with the teacher in the classroom; this will aim at strengthening or thoroughly analysing all that has been done with the classmates. In this sense, inclusion is not denied, yet it is reinforced and made appropriate to the subject involved.

The choice of the targets

In order to develop a learning process in the logical-mathematical sphere for the disadvantaged pupil (and for all pupils), it will be necessary to start from the evaluation of abilities and work according to his or her needs; moreover the syllabus will not have to be concerned just with the sharing out in cycles and single classes, yet it should imply a global growth project regarding the different thematic areas.

In our opinion, the choice of the targets and activities to be suggested to these pupils must be, first of all, a choice of priorities: it is necessary to single out the important and essential goals, together with the abilities these pupils are lacking, in order to avoid them and protect the global result.

The following tables compare some basic mathematical targets (italic: on the left) with some of the activities which make them real in everyday life (on the right). Mathematical targets are chosen thinking of 6-14 year old pupils.

TO RECOGNIZE, NAME, CLASSIFY	
• *To classify on the grounds of given attributes* • *To combine objects and attributes* • *To interpret the main logical connectives as set operations*	• To develop the senses and be able to distinguish and separate • To identify types of shops and products • To get one's bearings in the departments of a supermarket • To set the table, tidy up the room and put one's own things in order, etc. • To use the 'Yellow Pages' • To know how to look for books in a library

TO RELATE, ARRANGE, CREATE CORRESPONDENCES	
• *To discover rhythms and regularities in series of objects, images and sounds; to create vice versa successions according to given rules* • *To represent space / time successions, order relations and correspondences, all related to concrete situations* • *To compare concepts of relation, correspondence and function in different contexts*	• To be able to perceive and relate elementary visual and sound messages (doorbell, phone, switches, etc.) • To know how to deal with some social rules (to play, to dress oneself, etc.) • To listen to one's cardiac rhythm and reproduce musical rhythms • To use public transports following the series of stops • To understand the use of time sequences (morning, afternoon, evening) • To organize a standard day • To understand family ties • To use the phone and the telephone book • To prepare one's own schoolbag • To associate every pupil with his seat in the classroom, locker, etc.

TO HAVE A GOOD KNOWLEDGE OF THE CONCEPT OF NUMBER, KNOW HOW TO COUNT, PERFORM SIMPLE OPERATIONS	
• *To count on the number line (extended if possible) in a progressive and regressive way* • *To compare according to quantity* • *To read and write numbers* • *To make simple calculations, both mentally and written* • *To guess and know how to use the properties of the operations* • *To know the concept of fraction as part of a whole* • *To extend the concept of number: from natural to relative and rational numbers* • *Ratios, percentages and proportions* • *Multiples and divisors*	• To be able to count money • To use public transport • To follow street numbers • To do one's shopping • To be able to read thermometers • To be able to look at the calendar, the train timetable, etc. • To buy things when they are on sale • To cook following a recipe and make portions

 ISSN: 1463-6212
http://www.down-syndrome.info/library/periodicals/dsnu/04/01/

TO FOLLOW, SHOW, DRAW PATHS	
• *To move along set courses* • *To describe paths performed by other pupils* • *To train the mind to develop visuo-spatial skills*	• To orientate oneself in space • To get and give right information • To be able to help and get helped as for moving • To be able to "play" (obstacle race, videogames, etc.) • To arrange the furniture

TO BE ABLE TO CHOOSE THE OPERATIONS WITHIN PROBLEMATIC SITUATIONS	
• *To translate problems expressed through words into mathematical representations, being able to choose the suitable operations* • *To single out problematic situations within experience fields and advance hypotheses of solution* • *To identify data and meaningful variables in a problem* • *To solve problems using different processes*	• To organize shopping • To make travel plans • To organize a meal, a party, etc. • To be able to use the phone in different situations and react properly to unexpected events • To be able to use a vending machine properly, recognizing and using the instructions in sequence • To know how to use ordinary tools (remote control, videocassette recorder, washing machine, etc.) • To use a computer

MEASURES	
• *To know the main units of measurement and be able to use them properly* • *To choose suitable instruments in order to measure* • *Metric system*	• To be able to read a recipe • To be able to measure the height and weight of classmates • To be able to measure properly in order to perform simple tasks and activities in the classroom (locker's covering, curtains, etc.) • To build objects (kites, little houses for animals, etc.) • To arrange the furniture in the classroom

FORMS, FIGURES AND THEIR FEATURES	
• *To recognize the simplest kinds of geometrical plane and solid figures in the objects* • *To measure the areas and perimeters of the main plane figures* • *Study of the figures in a plane and in space starting from material models* • *Lengths, areas, volumes, angles and their measure*	• To be able to identify unknown objects, by means of the description of their shape • To recognize the use of certain objects on the grounds of their shape (pots and pans, kitchenware, work tools) • To recognize the most important road signs • To use jigsaws or games implying the creation of figures (for example Tangram, Lego, etc.) • To disassemble and reassemble objects • To decorate a room

TO WORK IN THE CARTESIAN PLANE	
• *To identify positions and movements within the plane and represent situations by means of grids with positive integer coordinates* • *Use of the coordinate methods in concrete situations*	• To play everyday games (for example battleships) • To be able to find a street or a path on a map • To be able to interpret and use a map

TO NOTICE THE (BASIC) TRANSFORMATIONS OF THE PLANE	
• *To identify symmetries within objects and figures* • *Enlargements and scale reduction* • *Shadows, representations in perspective, pictures, paintings*	• To be able to observe and understand shadows • To recognize the scheme of the body (right/left, forward/backward, etc.) • To be interested in art • To trace figures (also by using three-dimensional techniques) • To use the mirror and recognize the sizes of clothes • To be able to use a photocopier, a camera, etc.

 ISSN: 1463-6212
http://www.down-syndrome.info/library/periodicals/dsnu/04/01/

Learning evaluation

The division of objects and mathematical concepts suggested above is not meant to be rigid and hierarchical, because they constantly intertwine; some pupils will probably be able to get to a higher level in some of them and not in others, or vice versa.

Such a distinction can be useful to the teacher in order to show every pupil the way to get to the highest level; besides, the teacher must be aware that the achievement of each level is valid by itself and that every pupil is entitled to improve as much as possible in this itinerary within the class.

Moreover, it is necessary for the teacher to understand how the same ability can exist and be simultaneously valued on different levels, also taking into consideration the features of certain kinds of disabilities or the individual needs.

To get a better look of what our idea means, consider the following examples:

- in order to value the knowledge of measures and of simple geometrical figures, it will be useful to suggest the creation of a frame for the picture of the class, paying attention to the manual ability of the student and consequently graduating this activity accordingly (from the simple cutting of a cardboard support to the cutting and mounting of wooden sticks);
- in order to value the mastery of calculation abilities we may suggest simple problems of transaction in an active way (from buying a snack at the bar at school to the organization of a small party).

Both of these problems are apt to be developed on different levels or enriched with economical concepts.

Evaluation and self-evaluation

It is important to point out that the evaluation must take place within the activities of the class. If the teacher is aware that many features of autonomy are to be obtained through the achievement of mathematical abilities, then the evaluation of the pupils' competences regarding their autonomy will spontaneously allow the realization of their mathematical skills, involving the pupil as well. So, even the evaluation will represent a self-evaluation for the disabled pupil, a kind of self-discovery, a real instrument of personal growth; let's remember an idea, (due to A. Canevaro): the development of a self-evaluation, in order to help the pupil to be aware of his situation: the evaluation of the disabled student should be carried out twice; at first with **appropriate support**, and then without. In this way the child will be led to understand his performance with or without the support while, at the same time, there will be an encouragement of his social integration.

In order to get to a self-evaluation, the targets need to be stated, agreed upon and objectively related to common situations, which should be easy to interpret. This may seem easy for language learning, yet it seems difficult when dealing with mathematics. The evaluation will be easier for the teacher if the planning of activities is provided with an observation scheme, where every mathematical competence is pointed out and can be used by different operational suggestions: in this way, the teacher will be able to realize the abilities of the pupil at the end of his evaluation.

Conclusions

There are many examples in literature showing the advantages of mainstream school, involving mutual teaching and the concept of "zone of proximal development".[8]

Several methods we have drawn attention to and defined as extremely useful for learning, are the same for every kind of teaching and can be extended, in a more general way, to the mainstream school process of disabled pupils. These methods turn out to be useful to teaching, which is directed to the individuality of each pupil, fostering growth and taking account of each student's paths and rhythms.

Personal autonomy represents an important achievement for every young person, therefore the importance of the activities based on the above-mentioned methods is evident, since they are meaningful and useful to everybody. Once more, it is necessary to acknowledge the ability of mainstream school to reach unexpected skills: *even mathematics*!

References

1. Canevaro, A. (1992). In A. Contardi (1992). *Libertà Possibile.* Firenze: La Nuova Italia Scientifica.
2. Freudenthal, H. (1973). *Mathematics as an educational task*. Dordrecht: Reidel.
3. CREM (1995). *Les mathématiques de la maternelle jusqu'à 18 ans*. Nivelles: CREM.
4. Contardi, A., Pertichino, M. and Piochi, B. (1994). Apprendimento della matematica: insegnamento per problemi e alunni con handicap. *Psicologia e Scuola*, 71 (XV), 3-8.
5. Polya, G. (1957). *How to Solve It* (2nd ed.). Princeton: Princeton University Press.
6. Duncker, K. (1935). Zur Psychologie des produktiven Denkens [The psychology of productive thinking]. Berlin: Julius Springer.
7. Kanizsa, G. (1973). Il „problem-solving" nella psicologia della gestalt. In G. Mosconi e V. D'Urso. *La soluzione dei problemi.* Firenze: Giunti-Barbera.
8. Vygotskij, L.S. (1978). *Mind in Society.* Cambridge, Massachusetts: Harvard University Press.

Anna Contardi, Italian Association for Down Persons, Rome. annacontardi@hotmail.com; Michele Pertichino, Interuniversity Department of Mathematics, Bari. pertich@dm.uniba.it; Brunetto Piochi, Department of Mathematics "U. Dini", University of Florence, piochi@math.unifi.it.

 ISSN: 1463-6212
http://www.down-syndrome.info/library/periodicals/dsnu/04/01/